

Context of the Phd work in process and objectives to address during the internship

Introduction and overview of PhD thesis in process

Because of the importance of dairy industry in the Uruguayan economy, the complexity of dairy management systems and an increasing intensification process, it is interesting to study problems related to these systems using an operational research approach in order to solve practical agronomic problems.

An interesting option to obtain improvements in dairy industry is the application of optimization techniques to reduce costs, times and resource utilization levels. Optimization problems can be modeled using Mathematical Programming, which is divided into several branches according to the characteristics of variables and equations or inequalities that describe the model, for example linear or nonlinear programming problems. Another branch of Mathematical Programming is Combinatorial Optimization, which seeks to solve problems by doing an exploration of a discrete solution space. For these problems, the goal is to find an element (solution) of a (finite or infinite) discrete set which maximizes or minimizes a given objective function. There are two categories of methods for solving these problems, exact methods and heuristic methods (or approximate methods). Exact methods provide an optimal solution, while heuristic methods provide a good solution to the problem but not necessarily an optimal solution. Heuristic methods are used when the exact methods are not practical, for instance when their computation times or memory requirements are too large.

The goal of this work is to study problems related to milk production process by using operational research techniques that complement and enrich traditional agronomy approaches. Particularly, this work will be done within the context of a multi-institutional project called "Competitive, sustainable and simple milk production systems: the challenge of the Uruguayan dairy", which is funded by the Uruguayan research and innovation agency (ANII) within a call for Sectoral Innovation Networks. The University of the Republic (Faculty of Agriculture, Faculty of Veterinary and Faculty of Engineering), Conaprole (National Milk Producers Cooperative), INIA (National Institute of Agricultural Research), INALE (National Milk Institute) and CRI Lechero (Regional Consortium of Dairy Innovation, public-private partnership) are involved in this project. Considering the increasing complexity of dairy systems, part of this project is to address the problem of combining high production and persistence of pastures with efficiency production, in terms of human capital, productive and reproductive variables (including product quality), health and animal welfare. A program of research and technology transfer in the long term is being implemented. A sustainable intensification of production based on greater individual productivity and an increased stocking rate, with different feeding strategies during lactation over a productive and persistent forage base is considered.

The ANII project consists of four components. The first component, called "Feeding strategies: an integrated view of animal potential of dairy cows under grazing", aims to

study the impact of changes in nutrition and animal management on the different production systems. Different feeding strategies (experimentally and commercially) will be evaluated. The second component, called "Stocking rate: looking for the balance between biomass production and milk production considering the biotype and the region", aims to study (considering pasture level and system level production) the most important management factors in productivity and persistence of perennial pastures that affects the animal response. The third component, called "System reproductive and animal welfare indicators: dairy type, animal management and feeding as risk factors", seeks to establish the analytical and recording methodology that facilitates the association of variables and the processes understanding in order to conceptualize management recommendations to get more precision.

This thesis proposal is included in the context of the fourth component called "Analysis and re-design of production systems: integration of analytical and systemic investigation". The scope of this component is to integrate experimental information and real farm information (obtained from the remaining components of the project) in dynamic optimization models and simulation models. The goal of those models is to describe, analyze and re-design the milk production systems.

Currently there are different models available in the literature that tackle problems related to milk production systems. This thesis focus is to unify a number of these models from a tactical perspective, considering a multi-period approach (12 months), and integrating operational decisions using more detailed models (with monthly behavior).

The main idea is to develop two approaches. Firstly, we want to integrate optimization models and simulation models considering the characteristics data of a specific system (fields spatial data, herd size and composition, pastures available, etc. characteristics) in order to evaluate the performance of the system (by obtaining a solution close to optimal) covering 12 one-month stages. Secondly, we want to develop an optimization multi-objective evolutionary algorithm to generate an approximation of the Pareto front, taking as decision variables a sub-set of input parameters, and using different objective functions (maximizing total production, maximizing production efficiency, maximizing economic gain, minimizing capital invested, etc.).

Some of the problems we want to take into account in the thesis are the food resource allocation problem, the problem of reducing the cost of the herd diet, the problem of predicting the behavior of the systems when economical or environmental changes occur, the estimated live weight of the cows after having fed, the estimation of the pastures growth after grazing, among others.

The main idea is not to define one large model to cover all aspects mentioned above, but to develop a tool that allows the interaction between several modules each representing one sub-model or aspect of the system. We understand that defining a single integrated model may become impracticable, and its development will surely be very complex. However, if we address the problem as individual models, and find the way to make them interact, it will be easier to understand the tool, easier to extend and we will have the possibility to determine which modules to use before each execution of the main model.

The main model can be seen as a multi-stage optimization model. From resource allocation rules for a period of time, and the subsequent results of the overall process, we can create a system that uses those results as income data and then repeat the process throughout the year.

The food resource allocation problem was addressed by Gaston Notte in his MSc. thesis, and later an article was published (Notte et al., 2016, Resource allocation in pastoral dairy production systems: Evaluating exact and genetic algorithms approaches, *Agricultural Systems*). The dairy herd food resource allocation consists in determining how to allocate the available resources that are spatially distributed (pasture zone and feeding places) in order to maximize dairy production or the economic benefit (we refer to economic benefit as the margin over feeding cost). Those resources are different food activities located in field areas, associated with a certain availability of dry matter and energy, that must be allocated to different types of cows, so we need to determine where to distribute each cow (for feeding purposes) considering their characteristics. In other words, we need to group the cows and distribute them into different feeding areas. The problem solution includes as many groups of cows as existing feeding areas. Many combinations to assign those cows within existing resources can be done, but some solutions are better than others. This allocation process is usually based on the experience and intuition (and even traditions) of the producers, following some management rules considering parity, days in milk, actual milk production, among others, but this problem is difficult to solve when the problem size increases and/or when resources are scarce. A mathematical programming model was formulated and an exact method, as well as a Genetic Algorithm (GA) were used as resolution methods.

The problem of reducing the costs of the herd diet was addressed by Guillermo Battegazzore, and his MSc. thesis will be done in this context. This problem is presented as a combinatorial optimization problem which minimizes the annual costs considering biological constraints of cows and restrictions on the availability of food throughout the year. Particularly we understand that this problem and the food resource allocation problem can become one and can be presented as a single multi objective problem.

The problem of predicting the behavior of systems when economic or environmental changes occur can be addressed as a problem of predicting costs and production on the systems. In order to reduce those costs or maintain/increase the production, the study of possible alternatives is proposed.

Models that include production, economic and environmental aspects are not available in Uruguay. The examples reported in the international literature are few (and have very low applicability to the Uruguayan context).

To solve this problem we also consider the possibility of incorporating a financial model that allows producers to evaluate various alternative solutions.

The problem related to the estimated live weight of the cows after having fed can be easily found in the literature and can be adapted to our systems. This problem can be

defined as a single model and then be invoked after each stage of the resource allocation model.

Similarly to the previous case, the estimation of the pastures growth after grazing can be found in the literature and can be adapted to our systems. In this problem, balanced fertilization of pastures is one of the best tools to increase pastures supply, so their use will be considered.

Also, this problem may be defined as a single model and then be invoked from the main model.

After each stage of the process, considering the passage of time and the different strategies used to feed the herd, we have to use a model to update the changes in size and characteristics of the herd. This model must contain a sub-model that handles the animals being raised.

Materials and methods

A first approximation of the problem was based in the food resource allocation model, and was presented in terms of supply and demand.

The supply structure was defined by the availability of food resources, while the demand structure was defined by the energy required by the herd (based on the nutrient requirements of dairy cattle as published by the NRC (National Research Council, 2001)). The resource allocation model allows to group animals and move the cattle to a set of known field areas (pastures and/or feeding places). Considering each pasture activity or food type available for each feeding place, and depending on the different conditions presented by each animal to produce milk, the goal was to find a resource allocation by grouping cows and distributing these groups to the feeding areas, that maximizes the total milk production or the economic benefit.

The most important food supply of Uruguayan dairy production systems are pastures, which are located in different zones and are differentiated by the distance to the milking parlor and the herbage characteristics like mass (measured in kilograms of dry matter per hectare, kg DM/ha), energy density (measured like the net energy megacalories per lactation per kilogram of dry matter, Mcal ENL/kg DM) and cost (measured in United States dollars per ton of dry matter, US dollars/ton DM). In this work, the available pastures were considered as a finite resource, and the amount of pasture eaten by cows and the rate of regrowth was contemplated in order to calculate the next stage (month) availability.

We also considered different types of conserved forage and concentrates that differ in their energy density, availability and costs.

Food demand was determined by the specific features of each cow, considering the following attributes: body weight (*bw*, 500-600 kg), genetic potential (*gp*, 5500-9000 liters of milk in 305 days), lactation days (*ld*) or lactation weeks (*lw*), parity or lactation number (*ln*) and milk solids content like fat (*g*) and protein (*p*).

In this work, we considered animals with varying body weight and genetic potential, and we fixed (without loss of generality) the other parameters to the following values: $ld=140$ days, $lw=20$ weeks, $ln=1-5$, $g=3.6\%$, $p=3.1\%$.

These values were based on common values observed in farms. In spite of that, this methodology and model can accommodate any other values which better correspond to different situations.

The milk production for each cow at a given time was obtained by the amount of energy the animal has available.

The amount of available energy was calculated as the amount of acquired energy through food minus the energy requirements.

The acquired energy was calculated as the amount of DM in kilograms consumed by the animal multiplied by the energy value of the food source.

It is important to note that the potential consumption in kilograms per day was calculated for each type of animal, which influences the potential milk production. The potential consumption was used to limit the maximum consumption for each animal.

Each pastoral zone was associated with a specific feeding type, so the energy value acquired by each cow depends on the pastoral zone.

The energy requirements were the sum of the basal requirement and the movement requirement. The movement requirement was the energy cost of moving from the pasture to the milking parlor and from the milking room to the next destination.

If the animal was already in a feeding place, there is no movement requirement.

The liters of milk produced were calculated by dividing the available energy by the equivalent energy per liter (ENI).

Resource allocation problem formulation

The resource allocation problem consists in distributing cows into the field zones throughout the year, which is divided in twelve stages (month).

Every stage, the herd is divided in different batches (every batch is formed by grouping cows of different type). Taking into account the number of days for each month and considering two daily milkings, each batch is assigned to each field zone in several opportunities.

To determine the best animal distribution, it is necessary to know the amount of milk obtained for each distribution. The total milk production obtained by distribution was calculated as the sum of milk production obtained by each cow.

Great differences can be found between the original approach and this one. In the original one we did not consider batches, the distributions were done for each milking, we did not contemplate the regrowth of the pasture, we did not include some of the biological constraints that we considered in this approach, among others.

The resulting mathematical formulation of the problem is shown in Equations 1 to 15.

$$\max \sum_{s \in S} \sum_{b \in B} \sum_{z \in Z} \sum_{t \in T} ((w_{sbzt} * CL_z) - y_{sbz} * x_{sbt} (BR_t/2 + Cte * D_z * BW_t)) / ENI \quad (1)$$

/* objective function: maximize milk production */

st:

/*Resource allocation constraints */

$$\sum_{z \in Z} y_{sbz} = DA_s * 2 \quad \forall s \in S, \forall b \in B \quad (2)$$

/* the sum of assignments for each zone must be equal to the number of days multiplied by 2*/

$$\sum_{b \in B} x_{sbt} = C_t \quad \forall s \in S, \forall t \in T \quad (3)$$

/*for each type of cows, the sum of cows for each batch must be equal to the number of cows */

$$\sum_{t \in T} x_{sbz} \geq MinBS \quad \forall s \in S, \forall b \in B \quad (4)$$

/*the number of cows in each batch must be bigger than the minimum batch size */

$$\sum_{t \in T} x_{sbz} \leq MaxBS \quad \forall s \in S, \forall b \in B \quad (5)$$

/*the number of cows in each batch must be smaller than the maximum batch size */

$$y_{sbz} * x_{sbt} * PC_t / 2 \geq w_{sbzt} \quad \forall s \in S, \forall b \in B, \forall z \in Z, \forall t \in T \quad (6)$$

/*the potential consumption must be equal or bigger than the real consumption */

$$\sum_{b \in B} \sum_{t \in T} w_{sbzt} \leq v_{sz} - MG_{sz} * H_z \quad \forall s \in S, \forall z \in Z \quad (7)$$

/* the real consumption for each zone must be lower or equal than the available amount of food minus the minimum requirement for a controlled regrowth of the pasture */

/*Amount of food calculation for each stage */

$$v_{sz/s=march} = F_z * H_z \quad \forall z \in Z \quad (8)$$

/* initial amount of food */

$$v_{sz} = v_{s-1z} - \sum_{b \in B} \sum_{t \in T} w_{sbzt} + RG_{sz} * DA_s * H_z \quad \forall s \in S / s > 1, \forall z \in Z \quad (9)$$

/* the available food in each stage is calculated considering the food consumption in the previous stage and the growth rate per day */

/*Biological constraints */

$$\sum_{z \in Z} w_{sbzt} * CL_z \geq MinE_{st} * x_{sbt} * DA_s \quad \forall s \in S, \forall b \in B, \forall t \in T \quad (10)$$

/*total energy consumption must be equal or bigger than the minimum requirement*/

$$\sum_{z \in Z} w_{sbzt} * CL_z \leq MaxE_{st} * x_{sbt} * DA_s \quad \forall s \in S, \forall b \in B, \forall t \in T \quad (11)$$

/*total energy consumption must be lower or equal than the maximum requirement*/

$$\sum_{z \in Z} w_{sbzt} * P_z \geq MinP_{st} * x_{sbt} * DA_s \quad \forall s \in S, \forall b \in B, \forall t \in T \quad (12)$$

/*total protein consumption must be equal or bigger than the min. requirement*/

$$\sum_{z \in Z} w_{sbzt} * P_z \leq MaxP_{st} * x_{sbt} * DA_s \quad \forall s \in S, \forall b \in B, \forall t \in T \quad (13)$$

/*total protein consumption must be lower or equal than the max. requirement*/

$$\sum_{z \in Z} w_{sbzt} * NDF_z \geq MinNDF_{st} * x_{sbt} * DA_s \quad \forall s \in S, \forall b \in B, \forall t \in T \quad (14)$$

/*total NDF consumption must be equal or bigger than the min. requirement*/

$$\sum_{z \in Z} w_{sbzt} * NDF_z \leq MaxNDF_{st} * x_{sbt} * DA_s \quad \forall s \in S, \forall b \in B, \forall t \in T \quad (15)$$

/*total NDF consumption must be lower or equal than the max. requirement*/

In this model, each cow type was represented by the index t (in the set T), and each zone was represented by the index z (in the set Z).

To identify each batch the index b was added (in the set B).

Finally, each stage (month) was represented by the index s (in the set S).

As a consequence, x_{sbt} represents number of cows for each type for each batch, y_{sbz} represents how many times each batch is assigned to each zone, w_{sbzt} represents the total consumption of DM for each batch in each zone, and v_{sz} represents the available resources in each zone. A minimum level of herbage mass per hectare to ensure an adequate growth of pastures is considered.

This model assumes that the food resources available are shared uniformly between the cows assigned to a zone, so it is enough to know the whole zone consumption of DM and it is not necessary to represent the DM consumption for each cow.

Then the definition of the parameters and decision variables used are presented.

```

param S /* number of stages (months) */
param B /* number of batches */
param Z /* number of zones (pastures or feeding places) */
param T /* number of types of cows */

param CLz, z ∈ Z /* calories level for each zone */
param BRt, t ∈ T /* basal requirement for each type of cow */
param DIz, z ∈ Z /* distance from each zone to the milking parlor */
param BWt, t ∈ T /* body weight for each type of cow */
param ENl /* energy per liter */
param DAs, s ∈ S /* days in each stage (month) */
param Ct, t ∈ T /* number of cows of each type */
param PCt, t ∈ T /* potential consumption for each type of cow */
param MGsz, s ∈ S, z ∈ Z /* H0: minimum level of herbage mass per hectare */
param Fz, z ∈ Z /* initial amount of food per hectare in each zone */
param RGsz, s ∈ S, z ∈ Z /* rate of growth per day per hectare */
param Hz, z ∈ Z /* number of hectares per zone */
param MinEst, s ∈ S, t ∈ T /* minimum energy consumption per cow */
param MaxEst, s ∈ S, t ∈ T /* maximum energy consumption per cow */
param MinPst, s ∈ S, t ∈ T /* minimum protein consumption per cow */
param MaxPst, s ∈ S, t ∈ T /* maximum protein consumption per cow */
param MinNDFst, s ∈ S, t ∈ T /* minimum NDF consumption per cow */
param MaxNDFst, s ∈ S, t ∈ T /* maximum NDF consumption per cow */
param Pz, z ∈ Z /* amount of protein per kg of DM */
param NDFz, z ∈ Z /* amount of neutral detergent fiber per kg of DM */
param MinBS /* minimum batch size */
param MaxBS /* maximum batch size */

var xsbt, s ∈ S, b ∈ B, t ∈ T (integer)
/* number of cows for each type for each batch */
var ysbz, s ∈ S, b ∈ B, z ∈ Z (integer, >= 0)
/* how many times each batch is assigned to each zone */
var wsbzt, s ∈ S, b ∈ B, z ∈ Z, t ∈ T (>= 0)
/* total consumption of DM for each batch in each zone */
var vsz, s ∈ S, z ∈ Z (>= 0)

```


/* available resources in each zone */

Topics to cover during the internship

From the proposal presented in the Introduction section and the first approximation of the model shown in equations 1 to 15, some of the topics we would like to cover during the internship are as follow:

- Apply the first approximation of the model in a solver used in WUR.
- Run the model with Uruguayan real data to validate the model.
- if the model can not be solved in a quadratic way, try to linearize.
- Extend the model to a multi-objective model (max. the use of pastures, min. the use of concentrates (trade off), min. the cost of the diet, animal health, environmental objective, etc.).
- Define a metaheuristic to use.
- Define a second approximation of the model:
 - 1) change parameters for decision variables:
 - determine the pasture of each zone
 - determine the amount of food needed to obtain the potential production
 - determine the number of cows for each type
 - 2) incorporate infrastructure concepts
 - 3) incorporate financial concepts