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Resource allocation in pastoral dairy production systems: Evaluating exact and genetic algorithms approaches

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ABSTRACT

The problem of food resources allocation to a heterogeneous dairy herd was studied in this paper. We focused on how to allocate available resources by grouping cows and their subsequent distribution in the field (pasture and/or feeding area). The main goal of this paper was to maximize either milk production or the margin over feeding cost for the entire dairy herd. The input of energy from different feed resources and the animal requirements of energy were considered. A mathematical model and a Genetic Algorithm (GA) were programmed. An experimental evaluation was performed in order to analyze the quality solution of the GA and to study how the resource allocation should be performed by interpreting the solutions' structure for both methods. The diversity of the solutions provided by the GA was also studied. The experimental evaluation showed that the gap values (milk production difference) between the GA and the Exact Method (EM) solutions were smaller than 2%. Also, when food resources were scarce, there was a great difference (almost a 50% difference for a herd of 1500 cows) between the GA and the EM solutions' structure. The results showed that values obtained by the GA were very close to the values obtained by the exact method, but generating different assignment structures, presenting a good diversity and a wider exploration of the solutions' space.

1. Introduction

The agricultural industry is one of the most important sectors of the Uruguayan economy. In particular, the dairy industry represents 9.3% of the gross value of agricultural production, ranking third after beef and rice (Yavuz et al., 2010).

The Uruguayan area dedicated to dairy production is estimated in 857,000 ha (Uruguay, 2012). The dairy production exceeds 1800 million liters per year (Freiría and Hernández, 2010; Yavuz et al., 2010), being Uruguay the largest milk producer per capita in Latin America with about 550 l, and having one of the highest consumption rates in the world exceeding 250 l/person (Freiría and Martin, 2014). The export ratio is about 61.2% of total production, and is exposed to the volatility of the international market for dairy products (Freiría and Hernández, 2010).

Because of the importance of the dairy industry for the Uruguayan economy, the complexity of the dairy management systems and the increase of the intensification process (Chilibroste, 2011), it is of highly interest to study problems related to dairy systems using an operational research focus to enrich traditional agronomy approaches. Particularly, in this paper we address the problem of food resources allocation in pastoral based dairy systems.

The dairy production system in Uruguay is defined as a pastoral system with supplementation (Chilibroste et al., 2012). The feed supply structure is defined by pasture directly grazed by cows, conserved forage and concentrates. Concentrates are resources that can be easily purchased in the market. Food demands at a given time are defined by the number of milking cows and their characteristics, particularly their potential for milk production, based on their body weight, days in milk, and potential food consumption. In Uruguay, cows are milked twice a day which implies animals round movement from the feeding area to the milking parlor. The feeding area can be a grazing plot or a feeding place, where cows receive the available supplements that will be described later. The feeding places are located close to the milking parlor.

The food resource allocation to a dairy herd consists in determining how to distribute the available resources in order to maximize dairy production or the economic benefit (we refer to economic benefit as the margin over feeding cost for the entire system). Those resources are different food types located in field areas and associated with a certain availability of dry matter (DM)[1] and energy, that must be allocated to different types of cows, therefore we need to determine how to

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[1] The dry matter is the remaining part of a material after removing all the water possible.

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distribute each cow (for feeding purposes) considering their characteristics. In other words, we need to group the cows and distribute them into different feeding areas. The solution to the problem involves having as many groups of cows as existing feeding areas. Many combinations can be done to assign those cows to existing resources, but some solutions are better than others. This allocation process is usually based on the experience, intuition (and even traditions) of the producers, following some management rules considering parity, days in milk, actual milk production, among others. However, this problem is difficult to solve when the problem size increases and/or when resources are scarce.

The application of Operational Research (OR) in agriculture has been extensive and varied (Weintraub and Romero, 2006). However the literature review performed has shown that this study is original, specially considering how the problem have been addressed. One of the first successful mathematical programming applications in agriculture was proposed by (Waugh, 1951), who used linear programming models to determine the minimum cost of the livestock ration. The nutritional requirements were fulfilled. Since then (early 1950s), many farmers have relied on linear programming for an optimal design of cattle diets (Weintraub and Romero, 2006). Years later, a model considering the ingredients prices was studied in order to find the optimal mixing food. To address this problem various multiple criteria models have been formulated (e.g. (Czyzak and Slowinski, 1990; Neal et al., 1986; Rehman and Romero, 1984; Zhang and Roush, 2002)). These models involve different types of food to feed cows, including grass species that can be grazed directly, but also species that must be harvested mechanically, which adds an additional cost. (Neal et al., 2007) addressed the problem of determining the most cost-effective combination of forage species by developing a linear programming model. All these previous works focused on optimizing different aspects of dairy systems, but the main difference with our work is that they developed models to find solutions without considering differences for food allocation or animal groups. This is the significant contribution of this research (in addition to the methods evaluated).

In another line of work, (Anderson and Ridler, 2010) presented a model that incorporates the economic relationships of production factors on a 100 ha pastoral system. Linear programming was used in order to optimize the economic benefit, considering some limitations such as average milk production, the herd replacement rate, cows' death rate and the maximum number of lactations per cow. Total and individual milk production is also an area where operational research techniques have been used. (Dean et al., 1972) analyzed the possibilities of increasing the efficiency and profitability of milk production per cow. In their work, production functions and linear programming models were combined to develop a computer system capable of providing feeding programs that optimize feeding dairy cattle. (Ridler et al., 2001) used a linear programming model to integrate individual components of a pasture based system, with the objective of implementing a unique economic model to maximize farm profitability. The papers mentioned above include different aspects of the dairy systems, but an important difference with our proposal (apart from how the herd is handled) is the definition of the model. Some previous work define the milk production as a constant and then determine the livestock ration in order to maximize the economic benefit. We maximized the economic benefit by allocating the available resources to then obtain the total milk production as an output.

A different problem that includes optimization models in dairy systems is the study of the optimal replacement for dairy cows. Considering that milk production depends on several factors that vary over time, the objective of this problem was to determine the optimum replacement policy for dairy herds keeping total milk production relatively constant. (Kalantari et al., 2010) tackled this problem using dynamic programming. Later, (Doole et al., 2012, 2013) introduced a nonlinear programming model that incorporates several important processes in pastoral dairy production system. Finally, (Doole and Romera, 2013) incorporated major biophysical processes to their management model for pastoral systems that uses a nonlinear optimization model. Through an empirical application, advantages and disadvantages obtained by the use of a high stocking were discussed. There is a great difference between the models reviewed and our proposal. In some cases because they perform an optimization in an annual basis and they considered a fixed-size cattle with predetermined characteristics.

In general, the problems mentioned above did not consider the animal grouping and did not differentiate how cows of different types were fed.

The main goal of this work was to determine how to allocate the available resources (by considering the entire herd and distributing the cows into the different food types) in order to maximize dairy production or the economic benefit. The problem of food resources allocation to a heterogeneous dairy herd was studied and modeled as a combinatorial optimization problem. Because of the inherent difficulty, large-scale combinatorial optimization problems usually cannot be solved with traditional exact approaches. In this context, several metaheuristics have been applied to obtain good quality approximate solutions in a reasonable execution time. Genetic Algorithms (GA) (Goldberg, 1989; Talbi, 2009), which belong to the Evolutionary Algorithm family (Bäck, 1996), have proven to be flexible and robust methods for effectively solve complex optimization problems. For these reasons, a GA tailored for the food allocation problem was also developed and included in the experimental study. Once the resource allocation results were obtained, we studied the allocation by interpreting the solutions' structure. We also analyzed the quality of the solutions obtained by the GA and studied the diversity of these solutions.

2. Materials and methods

The problem was presented in terms of supply and demand. The supply structure was defined by the availability of food resources, while the demand structure was defined by the energy required by the herd (based on the nutrient requirements of dairy cattle as published by the NRC (National Research Council, 2001)). The resource allocation model allows to group animals and move the cattle to a set of known field areas (pastures and/or feeding places). Considering each pasture activity or food type available for each feeding place, and depending on the different conditions presented by each animal to produce milk, the goal was to find a resource allocation by grouping cows and distributing these groups to the feeding areas, that maximizes the total milk production or the economic benefit.

2.1. Milk production model

The most important food supply of Uruguayan dairy production systems are pastures, which are located in different zones and are differentiated by the distance to the milking parlor and the herbage characteristics like mass (measured in kilograms of dry matter per hectare, kg DM/ha), energy density (measured like the net energy megacalories per lactation per kilogram of dry matter, Mcal ENL/kg DM[2]) and cost (measured in United States dollars per ton of dry matter, US dollars/ton DM). In this work, the available pastures were considered as a finite resource, and could only be used once. Returning to the same pasture was not considered an option and therefore, the rate of regrowth was not contemplated. We also considered different types of conserved forage and concentrates that differ in their energy density, availability and costs.

Food demand was determined by the specific features of each cow. Let i correspond to an arbitrary identification assigned to each cow, where $i = 1, \ldots, M$ (M is the considered number of cows). The specific features of each cow have the following attributes: body weight (*bw*,

[2] One calorie is equal to 4.184 joules.

500–600 kg), genetic potential[3] (*gp*, 5500–9000 l of milk in 305 days), lactation days (*ld*) or lactation weeks (*lw*), parity or lactation number (*ln*) and milk solids content like fat (*g*) and protein (*p*). In this work, we considered animals with varying body weight and genetic potential, and we fixed (without loss of generality) the other parameters to the following values: $ld = 140$ days, $lw = 20$ weeks, $ln = 1$–5, $g = 3.6\%$, $p = 3.1\%$. These values were based on common values observed in farms. In spite of that, this methodology and model can accommodate any other values which better correspond to different situations.

The milk production (*MP*) for a cow *i* at a given time was obtained by the amount of energy the animal has available. The amount of available energy (*avEn*) was calculated as the amount of acquired energy through food minus the energy requirements. The acquired energy was calculated as the amount of DM in kilograms consumed by the animal multiplied by the energy value of the food source.

It is important to note that the potential consumption (*potCons*)[4] in kilograms per day was calculated for each type of animal, which influences the potential milk production (*potProd*). The *potCons* was used to limit the maximum consumption for each animal. Maximum DM intake is equal or lower than *potCons* which was calculated as indicated in Eq. (1).

$$potCons = \left(0,372 \times potProd + 0,0968 \times bw^{0.75}\right) \times \left(1 - e^{-0,192 \times (lw + 3,67)}\right) \quad (1)$$

Each pastoral zone was associated with a specific feeding type, so the energy value acquired by each cow depends on the pastoral zone.

The energy requirements were the sum of the basal requirement (*bReq*) and the movement requirement (*mReq*). The basal requirement depends directly on the body weight (*bw*) of the cow, and its formulation is shown in Eq.(2). The movement requirement was the energy cost of moving from the pasture to the milking parlor and from the milking room to the next destination. This formulation considered the moving distance (*dist*) in kilometers multiplied by 2 (from the milking parlor to the pasture and coming back) and the body weight, and is shown in Eq. (3). If the animal was already in a feeding place, there is no movement requirement.

$$bReq = 0,08 \times bw^{0.75} \quad (2)$$

$$mReq = dist \times 2 \times 0,00045 \times bw \quad (3)$$

The liters of milk produced were calculated by dividing the *avEn* by the equivalent energy per liter (*ENl*), as shown in Eqs. (4) and (5).

$$ENl = 0,0929 \times g + 0,0547 \times p + 0.192 \quad (4)$$

$$MP = \frac{avEn}{ENl} \quad (5)$$

2.2. Resource allocation problem formulation

In this work, the problem was formulated as a mathematical programming model. In order to find a solution two alternatives were presented: an exact method (EM) and a genetic algorithm.

The resource allocation problem consists in distributing cows into the field zones, so, for each distribution, each field zone has a group of cows. To determine the best animal distribution it is necessary to know the amount of milk obtained for each distribution. The total milk production obtained by distribution was calculated as the sum of milk production obtained by each cow. Then, the solution of the problem can be seen as the interaction of two models: the resource allocation model (cows distribution model), and the milk production model defined by the prediction equations of the NRC model presented in Section 2.1. To determine the best resource allocation for the entire herd, the resource allocation model uses the milk production model.

In the resource allocation model, cows with similar characteristics were grouped together (same potential consumption, similar weight, etc.) and then each group was assigned to a field area. The time was represented by considering several milkings. This model allows us to define as many milkings as we want. The resulting mathematical formulation is shown in Eqs. (6) to (9).

$$\max \frac{\sum_o \sum_z \sum_t \left(w_{ozt} \times cal_z - y_{ozt} \times (bReq_t + mReq_{zt})\right)}{ENl} \quad (6)$$

sa :

$$\sum_z y_{ozt} = M_t \quad \forall o \in O, \forall t \in T \quad (7)$$

$$\sum_o \sum_t w_{ozt} \le Food_z \quad \forall z \in Z \quad (8)$$

$$w_{ozt} \le y_{ozt} \times consPot_t \quad \forall o \in O, \forall z \in Z, \forall t \in T \quad (9)$$

In this model cows were assigned to a field (a given zone in the farm, in the set *Z*), and different cow types (in the set *T*) were considered. Each cow type was represented by the index *t*, and each zone was represented by the index *z*. To identify each milking considering two milkings a day over several days ($2 \times nbDays$), the index *o* in the set *O* is added (with $O = 1,2,\ldots,2 \times nbDays$).

As a consequence, $y_{ozt}\mathbb{N}$ represents the number of cows of type *t* assigned to the zone *z* in milking *o*, and $w_{ozt}\mathbb{R}$ is a variable that represents the total consumption of DM in zone *z*, for cows of type *t* in milking *o*. This model assumes that the food resources available are shared uniformly between the cows assigned to a zone, so it is enough to know the whole zone consumption of DM and it is not necessary to represent the DM consumption for each cow.

The objective function (in this case maximizing milk production) was computed by adding all the energy obtained by cows from their food consumption (w_{ozt}, multiplied by the calories level for each food type cal_z), and subtracting the group energy basal requirements ($y_{ozt} \times bReq_t$) and energy transportation requirements ($y_{ozt} \times mReq_{zt}$), which depend on the animal characteristics, and in the case of the transportation requirements, on the distance to each zone *z*. The restriction shown in Eq. (7) forces the total number of every type of cow in each milking to be equal to M_t (the number of cows for every type of cow). The restriction shown in Eq. (8) ensures that the food consumed in each zone does not exceed the available resources ($Food_z$). Finally, the restriction in Eq. (9) enforces actual food consumption of each cow not to exceed its potential consumption.

Table 1
Herd description.

Type	BW (kg)	MP (l)	Prop. (%)
T1	600	9000	50
T2	550	7000	30
T3	500	5500	20

Notes: Type = type of cows, BW = body weight per cow in kilograms, MP = milk production potential (or genetic potential, liters of milk per cow in 305 days), Prop. = proportion of the total herd size.

[3] The genetic potential of a cow is the number of liters of milk per lactation that the animal can generate in 305 days.

[4] The potential consumption is the maximum food consumption of kilograms of DM per day for a cow.

Table 2
Resource information.

Activity	Description	ED (Mcal ENl/kg DM)	Distance (km)	Availability (kg DM)	Price (US dollars)
Z1	Pasture	1.4	0.5	1100	20
Z2	Pasture	1.5	1.5	1800	20
Z3	Pasture	1.5	2.5	1800	20
Z4	TMR feeding area	1.65	0	4500	80
Z5	TMR feeding area	1.44	0	4500	60

Notes: Activity = food activity, description = description of the food activity, TMR = total mixed ration, ED = energy density measured like the net energy megacalories per lactation per kilogram of dry matter, distance = distance to de milking parlor, availability = availability of the activity.

2.3. Exact method

The mathematical formulation presented in Section 2.2 was solved using an exact method (EM) with the objective of obtaining the optimal solution for each scenario.

To program the EM, we used the GLPK linear programming package (for its acronym, GNU Linear Programming Kit) (Makhorin, 2000). This package was designed to solve large scale linear programming and mixed integer programming, among others. To solve those problems, GLPK uses different algorithms, including simplex method (Luenberger, 1989), interior point methods (Lustig et al., 1994), and branch and bound (Lawler and Wood, 1966).

2.4. Genetic algorithms

To implement a GA for solving the problem, it was necessary to define a suitable encoding definition (see (Goldberg, 1989; Notte, 2014; Talbi, 2009)). The encoding must represent a distribution of the herd animals for each milking for several days, considering different cow types. In this paper, we used a group encoding that can be represented by a cube formed by the number of milkings ("days × 2 rows"), the number of field zones and the number of cow types. Each cube cell corresponds to a field zone, a milking and a cow type, and its value represents the number of animals of this type that have to be moved to that zone in that milking. A major advantage of this encoding was its simplicity. Since there is no theoretical or empirical evidence that a crossover or mutation operator performs better than any other for every optimization problem, we decided to use the operators from the classical Simple Genetic Algorithm with minimal adjustments. Specifically, for recombination and mutation variations the classical "one-point crossover" and "swap mutator" were applied, respectively. In this encoding, a not feasible solution (generated using the operators aforementioned) was a possibility, since a solution could be created with an incorrect number of cows. To ensure feasibility, a correcting procedure was implemented as follows: the incorrect number of cows (cows missing or in excess) in each row was determined, then a random field area was selected, and finally the incorrect number of cows was added or removed (as appropriate) to the selected area. This process was repeated until the total number of cows in the row was correct.

2.5. Computational experiments

In order to reach the main goal of this work (how to allocate the available resources), and considering the importance for using alternative resolution methods in problems of large instances (in our case GA), we present the three research questions (RQs) studied in this paper. RQ1 concerns the solution quality and the solution structure of the GA when the objective is to maximize the milk production (energy perspective). Then, RQ2 deals with the solution quality and the solution structure of the GA when the objective is to maximize economic benefit (economic benefit perspective). Finally, another point of interest is to study the diversity of the solutions provided by the GA, so RQ3 is concerned with how good are the diversity of solutions provided by the GA (diversity perspective).

To answer these questions, three computational experiments based on real test data were done. The test data was based on real-life data prepared by one of the authors of this work, Pablo Chilibroste, whose research area is focused on dairy production systems. To answer RQ1, we ran an experiment and then compared the results obtained by the EM and the GA. One execution of the GA for each number of cows was done. The objective function in this experiment was to maximize milk production. To answer RQ2, we ran an experiment and then compared the results obtained by the EM and the GA, but in this experiment the objective function was to maximize the economic benefit. One execution of the GA for each number of cows was done. Finally, the last experiment was run in order to answer RQ3, 30 executions of the GA for each number of cows were done.

The execution platform was a virtual machine running on a PC with Intel (R) Core (TM) i5-2400 (3.10 GHz CPU with 4 cores and 6 MB of cache) processor and 4 GB of RAM. The virtual machine operating system was Windows XP. That virtual machine uses 50% of the physical machine processor and had allocated 1.5 GB of the total RAM.

The scenarios used were determined by the existing food activities and associated characteristics, and by the herd size and description.

The dairy herd description was defined by the body weight and the genetic potential of the cow. In the experiments, we considered three cow types (T1, T2 and T3) with different values for body weight and genetic potential. Type T1 were cows of 600 kg of *bw* and 9000 l of *gp*, T2 were cows of 550 kg of *bw* and 7000 l of *gp*, and T3 were cows of 500 kg of *bw* and 5500 l of *gp*. Of the total herd size considered in each experiment, 50% corresponded to the first type, 30% corresponded to the second type and the remaining 20% corresponded to the third type (leading

Table 3
Cattle distribution obtained by the EM, in an energy maximization context, for different herd sizes (50–1500).

	Z1			Z2			Z3			Z4			Z5				
#C	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	TMP	IP
50	0	0	0	0	0	0	0	0	0	25	15	10	0	0	0	1843	36.9
210	0	0	0	0	0	0	0	0	0	105	63	42	0	0	0	7741	36.9
290	0	0	0	67	1	3	0	0	0	78	86	55	0	0	0	10255	35.4
350	0	0	0	76	1	0	42	0	0	57	101	66	0	3	4	12093	34.6
560	0	0	0	76	1	0	76	0	0	70	53	104	58	114	8	18496	33.0
600	2	3	0	76	1	0	76	1	0	49	68	115	97	107	5	19707	32.9
700	44	1	3	77	0	0	77	0	0	149	106	0	3	103	137	20372	29.1
800	44	1	3	77	0	0	77	0	0	153	88	138	49	151	19	19041	23.8
1000	44	1	3	77	0	0	77	0	0	300	76	197	2	223	0	16378	16.4
1200	44	1	3	77	0	0	77	0	0	400	136	237	2	223	0	13715	11.4
1500	44	1	3	77	0	0	77	0	0	552	0	260	0	449	37	9721	6.5

Notes: #C = number of cows, Z1;Z2;Z3;Z4;Z5 = field zones, T1;T2;T3 = type of cows, TMP = total milk production in liters per day, IP = individual production (average milk production per cow in liters).

Table 4
Cattle distribution obtained by a single run of the GA (number of generations 500), in an energy maximization context, for different herd sizes (50–1500).

#C	Z1			Z2			Z3			Z4			Z5			TMP	IP	Gap
	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3			
50	0	0	0	0	0	0	0	0	0	25	15	10	0	0	0	1843	36.9	0.00
210	0	0	0	0	0	18	0	0	0	105	63	24	0	0	0	7657	36.5	1.09
290	0	0	0	22	27	28	0	0	0	123	60	9	0	0	21	10167	35.1	0.86
350	0	0	0	10	64	3	0	0	2	164	28	0	1	13	65	12045	34.4	0.40
560	0	0	21	32	34	9	0	56	26	130	48	14	118	30	42	18368	32.8	0.69
600	28	4	15	0	67	23	41	9	27	184	3	5	47	97	50	19586	32.7	0.61
700	47	0	0	16	60	15	77	0	0	72	85	103	138	65	22	20162	28.8	1.03
800	31	7	26	45	31	28	11	28	65	149	114	5	164	60	36	18987	23.7	0.29
1000	1	2	61	0	90	0	0	30	74	297	148	4	202	30	61	16346	16.4	0.20
1200	10	1	53	34	56	0	34	56	0	73	247	64	449	0	123	13686	11.4	0.21
1500	17	0	47	55	35	0	77	0	0	256	212	159	345	203	94	9706	6.5	0.16

Notes: #C = number of cows, Z1;Z2;Z3;Z4;Z5 = field zones, T1;T2;T3 = type of cows, TMP = total milk production in liters per day, IP = individual production (average milk production per cow in liters), Gap = difference in total milk production between GA and EM solutions (computed as a percentage).

to constant herd percentage compositions). The information mentioned above is summarized in Table 1.

All experiments included five activities, each one located in a specific field zone (Z1, Z2, Z3, Z4 and Z5). Three of these correspond to activities located in field zones with pasture. The first zone (Z1) was defined with an energy density of 1.4 Mcal ENl/kg DM and with 1100 kg DM available, while the remaining two pastures (Z2 and Z3) were defined with the same energy value, 1.5 Mcal ENl/kg DM and the same available resource amount (1800 kg DM). The distances between the milking parlor and pastures of type one, two and three were 0.5 km, 1.5 km and 2.5 km, respectively. Additionally, two activities that correspond to food concentrates available on feeding places were included in the scenarios, one (Z4) with high energy density (1.65 Mcal ENl/kg DM) and the other (Z5) with low energy density (1.44 Mcal ENl/kg DM). In both cases with the same availability (4500 kg DM). The distances between these feeding places and the milking parlor was considered 0 km. The information mentioned above is summarized in Table 2.

For the experiment where maximizing the economic benefit was the main goal, we defined the milk and food prices. We considered the milk price as 0.35 US dollars per liter. The price of the three pastures were defined as a 20% of the milk price per kilogram of DM, while the prices of activities with high energy density and low energy density were 80% and 60% of the milk price per kilogram of DM respectively.

For the execution of the experiments, one milking was considered. In this case, the number of decision variables was small, and could be inspected easily.

In order to answer RQ3, we needed to determine a measure of diversity. The measure considered for this diversity was the distance between the solutions obtained by the exact method and the genetic algorithm. To have a measure of the diversity, 30 executions of the genetic algorithm for each number of cows were done. From the obtained results the gap value and distance to the optimal solution was calculated.

In order to calculate the distance between the solutions from the exact method and the genetic algorithm, the Euclidean distance was computed, taking into account the difference between each of the components of the EM and the GA solutions. The general Euclidean distance formulation is presented in Eq. (10).

$$\sqrt{\sum_{o,z,t}\left(y_{ozt}^{EM}-y_{ozt}^{GA}\right)^2} \tag{10}$$

3. Results

In this section we present the results of the three experiments mentioned in 2.5.

In order to evaluate the performance of the GA model, the gap value was studied. The gap value allowed us to evaluate the GA quality solution by comparing the total milk production obtained with the GA and the EM (computed as a percentage). A small gap value indicates that the results were similar, so the GA solution was good. We considered that a gap value was small when the difference between results was lower than 5%.

3.1. RQ1 - energy perspective

Table 3 presents the distribution obtained by the EM for each cow type in the different field zones. Results showed that when herd size was small, all cows were assigned the food type with the highest energy density. The last food type used was the one with the lowest energy density. In zones with pasture almost all the cows corresponded to type T1. Most of the cows of types T2 and T3 were sent to zones Z4 and Z5 (food concentrates). Also, the total and individual milk production decreased when the herd size was bigger than 700 cows.

Table 5
Cattle distribution obtained by the EM, in an economic benefit maximization context, for different herd sizes (50–1500).

#C	Z1			Z2			Z3			Z4			Z5			TE	IE
	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3		
50	0	0	0	25	15	10	0	0	0	0	0	0	0	0	0	472	9.5
210	0	6	42	29	56	0	76	1	0	0	0	0	0	0	0	1930	9.2
290	1	52	2	68	7	4	74	0	4	2	28	48	0	0	0	2509	8.7
350	25	24	2	74	0	4	74	0	4	2	81	60	0	0	0	2928	8.4
560	41	1	7	74	0	4	77	0	0	28	166	30	60	1	71	4299	7.7
600	41	1	7	74	0	4	77	0	0	28	166	30	80	13	79	4547	7.6
700	44	1	3	77	0	0	77	0	0	149	106	0	3	103	137	4596	6.6
800	44	1	3	77	0	0	77	0	0	153	88	138	49	151	19	4130	5.2
1000	44	1	3	77	0	0	77	0	0	302	55	197	0	244	0	3198	3.2
1200	44	1	3	77	0	0	77	0	0	402	20	237	0	339	0	2266	1.9
1500	44	1	3	77	0	0	77	0	0	552	0	260	0	449	37	868	0.6

Notes: #C = number of cows, Z1;Z2;Z3;Z4;Z5 = field zones, T1;T2;T3 = type of cows, TE = total earnings in US dollars, IE = individual earnings (average earnings per cow in US dollars).

Table 6
Cattle distribution obtained by a single run of the GA (number of generations 500), in an economic benefit maximization context, for different herd sizes (50–1500).

	Z1			Z2			Z3			Z4			Z5					
#C	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	TE	IE	Gap
50	0	0	0	25	15	10	0	0	0	0	0	0	0	0	0	472	9.5	0.00
210	9	12	26	42	24	11	53	21	3	1	6	2	0	0	0	1907	9.1	1.18
290	44	2	1	38	15	24	24	45	8	39	25	24	0	0	1	2461	8.5	1.93
350	12	24	10	5	41	40	63	14	0	95	25	19	0	1	1	2878	8.2	1.72
560	17	21	9	0	62	27	54	23	0	63	51	75	146	11	1	4231	7.6	1.57
600	1	52	2	27	50	0	10	62	17	131	12	49	131	4	52	4476	7.5	1.58
700	2	26	36	67	23	0	9	81	0	195	0	0	77	80	104	4586	6.6	0.22
800	26	8	33	6	58	42	77	0	0	150	70	56	141	104	29	4122	5.2	0.21
1000	4	4	60	78	0	0	33	33	38	65	234	53	320	29	49	3184	3.2	0.44
1200	13	0	54	77	0	0	77	0	0	154	142	110	279	218	76	2265	1.9	0.06
	24	33	8	2	37	65	2	20	82	186	221	130	536	139	15	853	0.6	1.74

Notes: #C = number of cows, Z1;Z2;Z3;Z4;Z5 = field zones, T1;T2;T3 = type of cows, TE = total earnings in US dollars, IE - average earnings per cow in US dollars, Gap = difference in total earnings between GA and EM solutions (computed as a percentage).

Table 4 presents the distribution obtained by the GA for each cow type in the different field zones. The gap value was very small but the distributions (structure of solutions) were different. Unlike Table 3, these solutions include cows of types T2 and T3 in field zones with pasture. There was no correlation between cow types and field zones.

3.2. RQ2 - economic benefit perspective

Table 5 presents the distribution obtained by the EM for each cow type in the different field zones. Results showed that when the herd size was small, the algorithm tried to avoid the zones with concentrates. The last food type used was the one in zone Z4 (high price for a low energy density). In zones with pasture almost all the cows corresponded to type T1. Most of the cows of types T2 and T3 were sent to zones Z4 and Z5 (food concentrates). Also, the total and individual earnings decreased when the herd size was bigger than 700 cows.

Table 6 presents the distribution obtained by the GA for each cow type in the different field zones. The gap value was very small but the distributions (structure of solutions) were different. Unlike Table 5, these solutions include much more cows of types T2 and T3 in field zones with pasture.

3.3. RQ3 - genetic algorithm diversity

In order to study the diversity of the GA, we calculated the gap value and the Euclidean distance between the solutions from the exact method and the genetic algorithm. From the 30 executions of the genetic algorithm for each number of cows, the solution with the lowest gap value, the solution with the lowest distance percentage, and the solution with the biggest distance percentage were obtained.

For each number of cows, a comparison of the total milk production between the solutions with the lowest gap value, lowest distance and biggest distance are presented in Fig. 1. Also, a comparison of the distance is shown in Fig. 2. Finally, a comparison of the gap value is presented in Fig. 3.

Additional data related to the GA diversity results can be found in the Appendix A. Table A1 presents the average and standard deviation of the solution and the gap and the distance percentage for each number of cows considered (over 30 different GA runs). Table A2 presents three solutions chosen over 30 different GA runs for each number of cows considered: the solution with the lowest gap value, the solution with the lowest distance percentage, and the solution with the biggest distance percentage. After calculating the distance, the distance percentage is computed as a percentage of the number of cows considered.

4. Discussion

4.1. RQ1 - energy perspective

The values presented in Table 3 indicate that solutions were made following a clear strategy: distributing as many cows as possible into field zones with higher energy density. The algorithm looked for

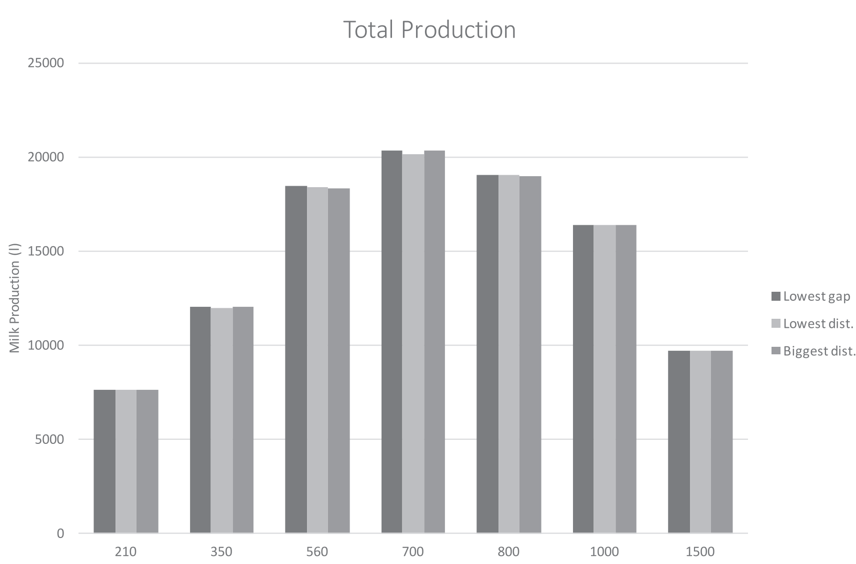

Fig. 1. Comparison of total milk production between the solutions with the lowest gap value, lowest distance and biggest distance.

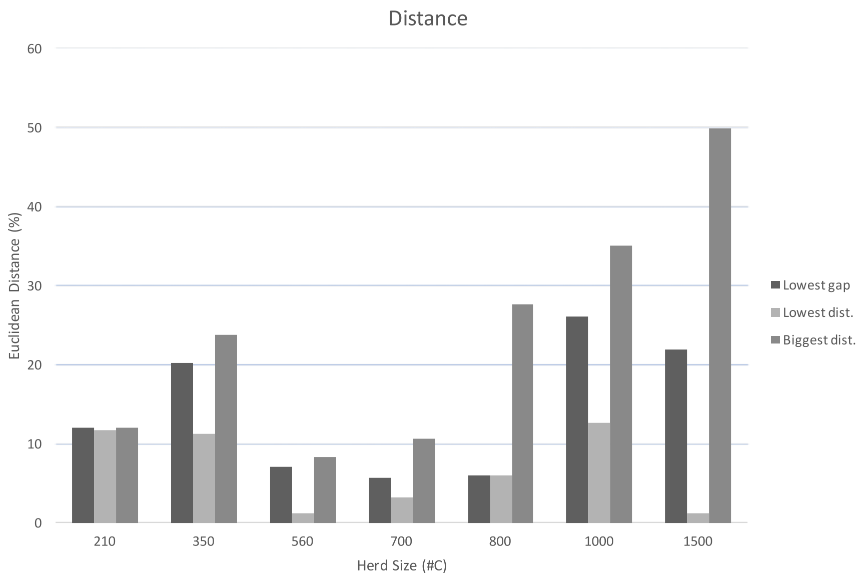

Fig. 2. Comparison between the solutions with the lowest gap value, lowest distance and biggest distance of the distance to the solution found by the exact method.

solutions where each cow consumed as much food as its potential consumption. As a consequence, the first 210 cows were sent to zone 4, but if the number of cows was larger (exceeding the availability of zone 4 to give each cow its potential consumption), the available resources in zone 2 were also used. If the herd size was even greater, available resources in zones 3, 5 and 1 were used in the mentioned order. When the herd size was over 700 animals, resources were not enough to feed all the cows with as much food as their potential consumption, and therefore sharing out the food was needed.

It was interesting to analyze the total milk production behavior and how the average individual milk production decreased once the food resources were not enough to feed all the cows with as much food as their potential consumption. Food resources were enough when considering herds of up to 700 cows, so each cow real consumption was almost equal to its potential consumption. In these cases the total milk production was higher than the production obtained by smaller herds. When the herd size was greater than 700 cows, the total milk production decreased, since the available food was shared among all the cows. Individual production reached its maximum value when the herd was up to 210 cows, where animals obtained the highest possible energy density. The results showed that until then all animals were sent to zone 4 (which provides higher energy density). When the herd size was increased, the EM distributed cows in other field zones, so the average energy acquired by the herd was smaller. A smaller amount of energy affected the average individual production causing a decrease in the total milk production. When herd size was over 700 cows, resources became scarce and the individual production average was notably affected.

The values presented in Table 4 showed that solutions were made following a similar strategy to the EM strategy. When herd size was small, cows were sent to zone 4. When herd size was greater, zone 2 was also used. If the herd size was much larger, the use of zones did not follow such a strong pattern as solutions presented by the EM. Despite the herd size, the number of cows in the EM solutions for zones 1, 2 and 3 remained constant, even for each cow type. Some variations were found for zones 4 and 5. However, the number of cows by zone or by type in GA solutions presented many changes, so there was good diversity.

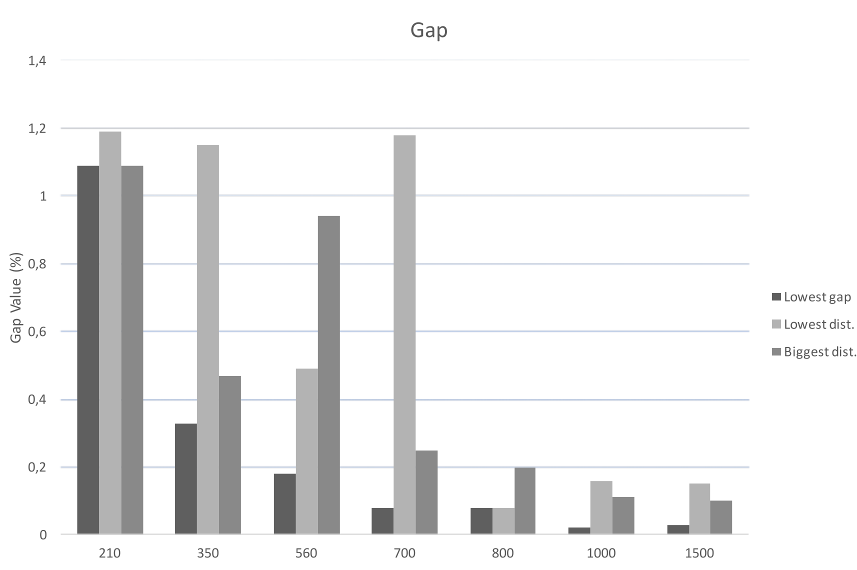

Fig. 3. Comparison of gap value between the solutions with the lowest gap value, lowest distance and biggest distance.

Total and individual milk production values and behavior were similar compared to those observed in EM solutions. The gap values showed that GA solutions were almost optimal. The biggest difference was observed when the herd size was 210, where the gap value reached 1,09%. If the GA maintains good diversity of solutions, an expert will have the possibility to choose between solutions near the optimum value with different assignments of cows to field zones.

Because we did not find this exact problem in the literature review, it was not possible to compare the results with previous work, but milk production values obtained from the GA were similar to those obtained with the EM, and this was expected because of the versatility of the GA for solving problems related to searching, optimization and machine learning (Goldberg, 1989).

4.2. RQ2 - economic benefit perspective

The values presented in Table 5 indicate that solutions were constructed following a clear strategy: distributing as many cows as possible into field zones with the best relationship between energy density and feed cost. Also, it was ensured that each cow consumes as much food as its potential consumption.

When the herd size was 50, cows were sent to zone 2, but if the number of cows was larger (about 210 cows), the available resources in zones 1 and 3 were also used. The information above indicates that the method was trying to pay as little as possible for food resources. If the herd size was even greater, available resources in zones 4 and 5 were used in that order. As well as in Section 4.1, when the herd size was over 700 animals, resources were not enough to reach their potential consumption, and therefore sharing out the food was needed.

When the food resources were not enough to feed all the cows with as much food as their potential consumption, the total earnings and the average individual earnings decreased. When considering herds of 700 cows, each cow real consumption was almost equal to their potential consumption, and the total earnings were higher than earnings obtained by smaller herds. When the herd size was greater than 700 cows, the total earnings decreased. The individual earnings reaches their maximum value when the herd was up to 50 cows, where animals obtained the best and cheapest feed considering the relationship between energy density and feed cost. The results showed that until then all animals were sent to zone 2 (which provides the best relationship between energy density and feed cost). The method used started distributing cows in other field zones when the herd size was increased, and that caused a decreased in the individual earnings of the herd. When the herd size was over 700 cows, resources became scarce and individual earnings were notably affected.

The values presented in Table 6 show that GA solutions were constructed following a similar strategy to the EM strategy. When the herd size was small, cows were sent to zone 2. When the herd size was greater, zones 1 and 3, and even zone 4 were also used. If the herd was much larger, the use of zones did not follow such a strong pattern as solutions presented by the EM. Despite the herd size, the number of cows in the EM solutions for the zones 1, 2 and 3 remained constant, even for each cow type. Some variations were found for zones 4 and 5. However, the number of cows by zone or by type in GA solutions presented many changes, so the GA had good diversity of solutions.

Total and individual earnings values and behavior were similar compared to those observed in EM solutions. The gap values indicated that GA solutions were close to the optimum value obtained by the EM. The biggest difference was observed in the case with 290 cows, where the gap value reached 1,93%.

Because we did not find this exact problem in the literature review, it was not possible to compare the results with previous work. This case was similar to the energy perspective, where values obtained from the GA were close to those obtained by the EM.

4.3. RQ3 - genetic algorithm diversity

Another point of interest was to discuss the diversity provided by the genetic algorithm. A measure of this diversity was the distance between the solutions obtained by the exact method and the genetic algorithm. To measure diversity, 30 executions of the genetic algorithm for each number of cows were performed, and the results obtained were reported in Appendix A (Table A2).

The results obtained by the genetic algorithm presented in Fig. 1 indicate that solutions with smallest and largest distances to the optimal solution found by the exact algorithm had similar gap values. Also, the solution with the lowest gap value was not related to the solution with the lowest distance (see Fig. 2). In some cases, the solution with the lowest gap value had a small distance, while in other cases it had a big distance, so that gap and distance values were not necessarily correlated. This also showed that the genetic algorithm reached a considerable degree of diversity in its solutions. Fig. 3 showed that the gaps values were all small (lower than 2%), so the solution quality was generally good, with high diversity. Fig. 3 also showed that the biggest gap difference between solutions was presented when the herd size was up to 700. When the herd size was bigger than 700 the gap values were very similar.

By taking advantage the good diversity of the genetic algorithm, we may help producers when making decisions, because it is possible to group cows in different ways and still maintain a good production or total profit. From the results obtained, it is possible to perform groups that differ considerably in the number of cows from different types and still maintain solutions near the optimum value (based in the gap values obtained).

5. Conclusions

Experiments confirm that the GA was well adapted to the problem, where values obtained were very close to those obtained by the EM. The results confirm that the GA reached high numerical solutions, obtaining results similar to those found by the exact method. Another interesting aspect of the GA is that it can maintain a good diversity of solutions, meaning that the producer can choose good quality feeding strategies with different assignments which can have other desirable properties not necessarily provided by the exact method solutions.

We analyzed the solutions obtained and found that the exact method solutions were built distributing as many cows as possible to field zones with higher energy density. The obtained results suggest that the animal real consumption must be equal to the potential consumption, otherwise serious losses will occur in economic earnings or milk production process efficiency. When the food resources available were not enough to fulfill the potential consumption of all cows, sharing out those resources was needed. If keeping the entire herd is not a constraint, the best option is not to share the available resources among the whole herd, this means to stop feeding a set of cows (probably meaning selling or disposing these animals, who would otherwise die).

It is also important to mention that the presented model only found the distance (between zones) to be a relevant factor when two food types had the same energy density or the same relationship between energy density and feed cost. These conclusions should be considered only for the specific scenario studied in this paper, and can change when using other data instances. Nevertheless, these conclusions show the potential of the model when analyzing a given situation.

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Appendix A. Additional data

Table A1
Average and standard deviation for distance between the solutions obtained by the EM and the GA.

#C	SolAvg (l)	SolSDev	GapAvg	GapSDev	DistAvg	DistSDev
210	7652	1.15	12.02	5.40	0.07	0.15
350	12011	0.68	18.68	31.80	0.26	3.63
560	18372	0.67	7.07	51.35	0.28	1.83
700	20210	0.79	8.35	100.86	0.50	2.05
800	19004	0.19	14.93	12.10	0.06	4.23
1000	16342	0.22	23.15	14.35	0.09	6.53
1500	9690	0.31	14.75	13.89	0.14	13.16

Notes: #C = number of cows, SolAvg = solution average in liters, SolSDev = solution standard deviation, GapAvg = gap average computed as percentage, GapSDev = gap standard deviation, DistAvg = distance average computed as percentage, DistSDev = distance standard deviation.

Table A2
Distance results by considering three solutions of the GA for each herd size: the solution with the lowest gap value, the solution with the lowest distance percentage, and the solution with the biggest distance percentage.

#C	Z1			Z2			Z3			Z4			Z5				Gap	Dist
	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	Tot		
210	0	0	0	0	0	18	0	0	0	105	63	24	0	0	0	7657	1.09	12.1
	0	0	0	0	10	7	0	0	0	105	53	34	0	0	1	7649	1.19	11.8
	0	0	0	0	0	18	0	0	0	105	63	24	0	0	0	7657	1.09	12.1
350	0	0	0	0	76	13	7	2	0	167	25	0	1	2	57	12054	0.33	20.3
	0	0	0	51	3	22	45	0	13	79	98	15	0	4	20	11954	1.15	11.3
	0	0	0	18	42	17	0	6	0	157	34	1	0	23	52	12037	0.47	23.8
560	4	0	0	0	88	0	0	0	86	192	0	0	84	80	26	18462	0.18	7.1
	0	0	0	33	38	6	72	0	1	0	129	95	175	1	10	18405	0.49	1.3
	3	0	27	51	2	22	64	3	8	99	63	30	63	100	25	18323	0.94	8.4
700	41	14	0	77	0	0	17	73	0	6	123	140	209	0	0	20356	0.08	5.7
	46	1	1	27	47	16	69	0	8	112	37	111	96	125	4	20131	1.18	3.3
	16	25	24	46	10	48	1	35	68	195	0	0	92	140	0	20322	0.25	10.6
800	6	53	0	78	0	0	0	90	0	99	78	160	217	19	0	19026	0.08	6.1
	6	53	0	78	0	0	0	90	0	99	78	160	217	19	0	19026	0.08	6.1
	31	0	40	78	0	0	30	16	58	201	0	0	60	224	62	19002	0.20	27.7
1000	48	0	0	80	0	0	77	0	0	85	159	143	210	141	57	16374	0.02	26.1
	10	47	0	77	0	0	67	24	0	180	178	115	166	51	85	16353	0.16	12.7
	9	0	56	64	26	0	78	0	0	192	118	0	157	156	144	16360	0.11	35.1
1500	56	0	0	77	0	0	77	0	0	54	450	72	486	0	228	9718	0.03	21.9
	32	24	0	87	0	0	77	0	0	505	182	122	49	244	178	9707	0.15	1.3
	42	21	0	81	0	0	77	0	0	11	149	113	539	280	187	9711	0.10	49.9

Notes: #C = number of cows, Z1;Z2;Z3;Z4;Z5 = field zones, T1;T2;T3 = cow types, Tot = total milk production, Dist = solution distance percentage, Gap = difference in total milk production between GA and EM solutions (computed as a percentage).

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